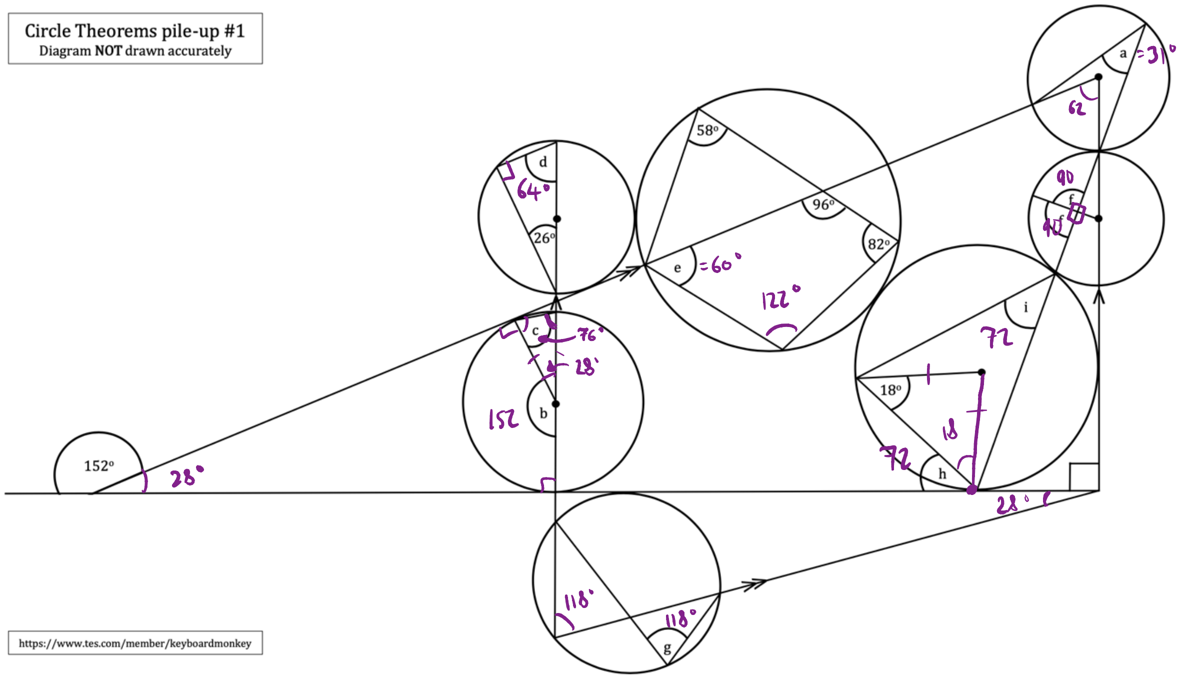
Circle Theorems (5) Geometric Reasoning

Do now:



Example

In triangle BCD, BC = BD.

ABC is a straight line.

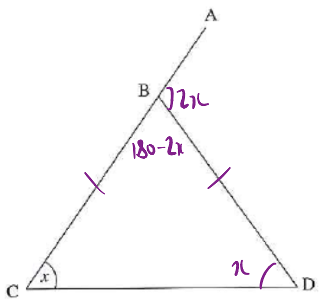


Figure 6.7

Prove that angle ABD = 2x.

$\angle BCD = x$ (BASE ANGLES IN AN ISOSCELES ARE EQUAL)

$\angle CBD = 180 - 2x$ (ANGLES IN A TRIANGLE SUM TO 180°)

$\angle ABD = 180 - (180 - 2x)$ (ANGLES ON A STRAIGHT LINE SUM TO 180°)

$= 2x$

AP is a tangent that touches the circle at P.

AP is parallel to QR.

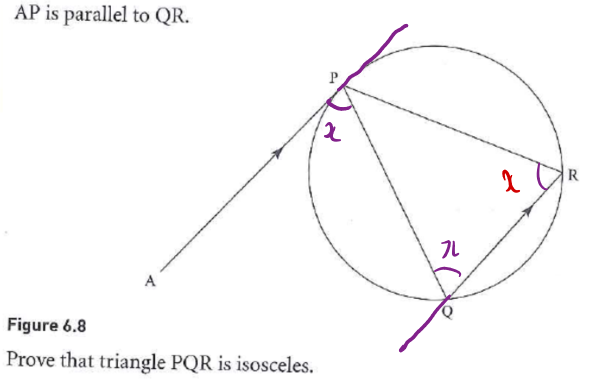


Figure 6.8

Prove that triangle PQR is isosceles.

LET $\angle APQ = x$

$\angle PQR = x$ (ALTERNATE ANGLES ARE EQUAL).

$\angle PRQ = x$ (ALTERNATE SEGMENT THEOREM)

Task

PQRS is a cyclic quadrilateral.

C is the centre.

Angle QPS = y Angle QCR = $2x$ Angle SQR = 40°

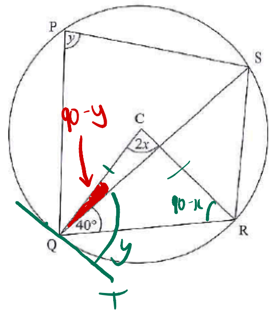


Figure 6.9

Prove that $y = x + 40$.

$\angle CRQ = 90 - x$ (Base angles in an isosceles triangle are equal).

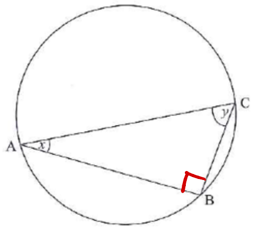
$\angle SQT = y$ (Alternate segment theorem).

$\angle CQS = 90 - y$ (Tangents and radii meet a right angles)

$90 - y = 90 - x - 40$

$x + 40 = y$

1 AC is a diameter. B is a point on the circumference.



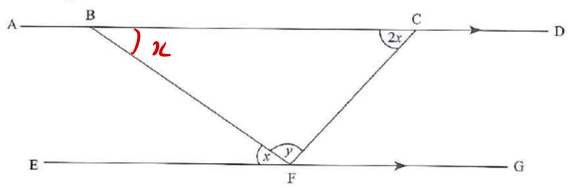
Prove that $x = 90 - y$.

$\angle ABC = 90°$ (Angles formed in a semi-circle are 90°)

$x + y + 90 = 180$ (Angles in a triangle sum to 180°)

$x = 90 - y$

2 ABCD is parallel to EFG.

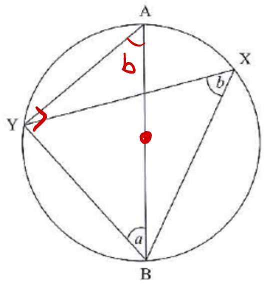


Prove that $3x + y = 180$.

$\angle CBF = x$ (Alternate angles are equal)

$3x + y = 180$ (Angles in a triangle sum to 180°).

3 AB is a diameter. X and Y are points on the circumference.



Prove that $a + b = 90$.

$\angle BAY = b$ (Angles in the same segment are equal)

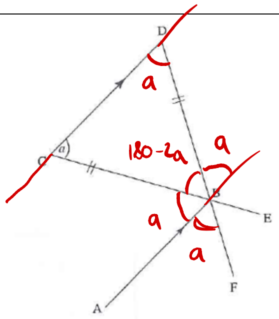
$\angle AYB = 90°$ (Angles formed in a semi-circle are 90°)

$a + b = 90°$ (Angles in a triangle sum to 180°).

4 CBE and DBF are straight lines.

CD is parallel to AB.

BC = BD



Prove that angle ABC = angle ABF.

$\angle CDB = a$ (Base angles in an isosceles triangle are equal).

$\angle DBC = 180 - 2a$ (Angles in a triangle sum to $180°$)

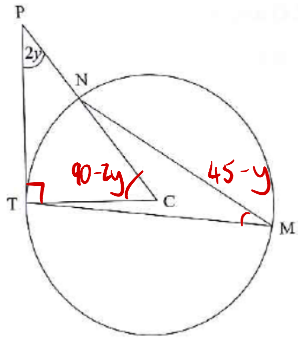
$\angle ADC = a$ (Alternate angles are equal)

$\angle OBE = a$ (Angles on a straight line sum to $180°$)

$\angle ABF = a$ (Vertically opposite angles are equal).

5 PT is a tangent, touching the circle at T. C is the centre.

M and N are points on the circumference.



Prove that angle TMN = $45 - y$.

$\angle PTC = 90°$ (Tangents and radii meet at right angles)

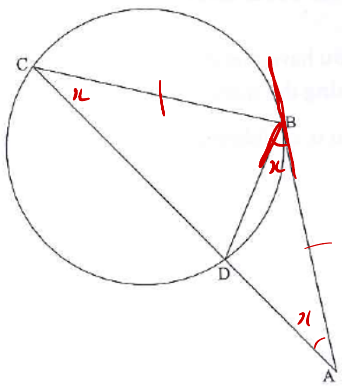
$\angle PCT = 90 - 2y$ (Angles in a triangle sum to $180°$)

$\angle TMN = 45 - y$ (Angles at the centre are double those at the circumference.)

6 AB is a tangent, touching the circle at B.

ADC is a straight line.

AB = BC



Prove that triangle ABD is isosceles.

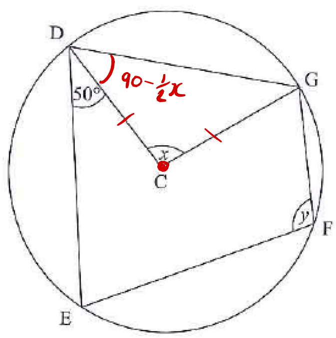
Let $\angle ABD = x$

$\angle BCD = x$ (Alternate segment theorem)

$\angle BAD = x$ ($\angle BCD = \angle BAD$ base angles in an isosceles triangle are equal).

7 DEFG is a cyclic quadrilateral.

C is the centre.



Prove that $x = 2y - 80$.

$\angle CDG = 90 - \frac{1}{2}x$ (Base angles in an isosceles triangle are equal).

$180 - y = 50 + 90 - \frac{1}{2}x$

$40 - y = -\frac{1}{2}x$ (Opposite angles in a cyclic quadrilateral sum to $180°$)

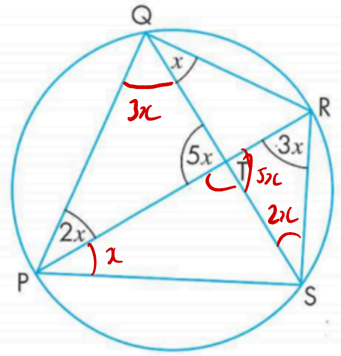
$y - 40 = \frac{1}{2}x$

$2y - 80 = x$

PQRS is a cyclic quadrilateral. PR and QS meet at T.

a Work out the value of x.

b Show that the angles of the quadrilateral and angle STP form a number sequence.



a.) $10x = 180$

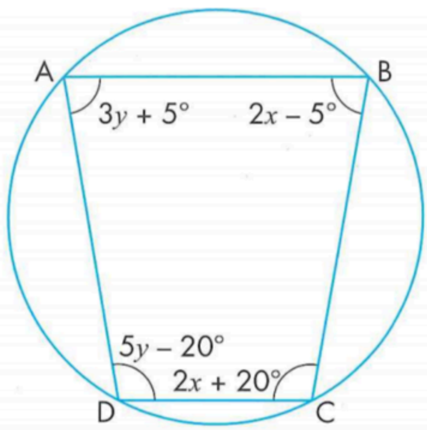
$x = 18°$

b.) $54°, 72°, 90°, 108°, 126°$

ARITHMETIC SEQUENCE $\quad U_n = 18n + 36$

ABCD is a cyclic quadrilateral.

Work out the values of x and y.



$2x - 5 = 180 - (5y - 20)$

$2x - 5 = 200 - 5y \quad (1)$

$2x = 205 - 5y$

$2x + 20 = 180 - (3y - 5)$

$2x + 20 = 185 - 3y \quad (2)$

$2x = 165 - 3y$

$205 - 5y = 165 - 3y$

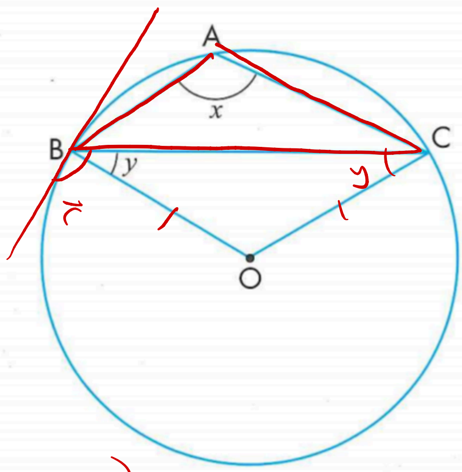
$40 = 2y$

$20 = y$

$x = 52.5°$

On the diagram, O is the centre of the circle.
Angle BAC = x and angle CBO = y.

Prove that $y = x - 90°$, giving reasons in your working.



$\angle OCB = y$ (BASE ANGLES IN AN ISOSCELES TRIANGLE ARE EQUAL).

$\angle CBZ = x$ (ALTERNATE SEGMENT THEOREM).

$\angle CBZ = y + 90$ (TANGENTS AND RADII MEET AT RIGHT ANGLES)

$\therefore x = y + 90 \rightarrow y = x - 90$